

NCERT Solutions Class 11 Maths Chapter 2 Relations and Functions

Question 1:

If $\left(\frac{x}{3}+1, y-\frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y .

Solution:

It is given that $\left(\frac{x}{3}+1, y-\frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$.

Since the ordered pairs are equal, the corresponding elements will also be equal.

Therefore, $\frac{x}{3}+1 = \frac{5}{3}$ and $y-\frac{2}{3} = \frac{1}{3}$

$$\frac{x}{3}+1 = \frac{5}{3}$$

$$\Rightarrow \frac{x}{3} = \frac{5}{3} - 1$$

$$\Rightarrow \frac{x}{3} = \frac{5-3}{3}$$

$$\Rightarrow \frac{x}{3} = \frac{2}{3}$$

$$\Rightarrow x = 2$$

$$y - \frac{2}{3} = \frac{1}{3}$$

$$\Rightarrow y = \frac{1}{3} + \frac{2}{3}$$

$$\text{and } \Rightarrow y = 1$$

Hence, $x = 2$ and $y = 1$

Question 2:

If the set A has 3 elements and the set $B = \{3, 4, 5\}$, then find the number of elements in $(A \times B)$?

Solution:

It is given that set A has 3 elements and the set $B = \{3, 4, 5\}$

Number of elements in set A , $n(A) = 3$

Number of elements in set B , $n(B) = 3$

Number of elements in $(A \times B) = (\text{Number of elements in } A) \times (\text{Number of elements in } B)$

$$\begin{aligned}
 n(A \times B) &= n(A) \times n(B) \\
 &= 3 \times 3 \\
 &= 9
 \end{aligned}$$

Thus, the number of elements in $(A \times B)$ is 9.

Question:3

If $G = \{7, 8\}$ and $H = \{5, 4, 2\}$, find $G \times H$ and $H \times G$.

Solution:

It is given that $G = \{7, 8\}$ and $H = \{5, 4, 2\}$

We know that the Cartesian product $P \times Q$ of two non-empty sets P and Q is defined as

$$P \times Q = \{(p, q) : p \in P, q \in Q\}$$

Therefore,

$$G \times H = \{(7, 5), (7, 4), (7, 2), (8, 5), (8, 4), (8, 2)\}$$

$$H \times G = \{(5, 7), (5, 8), (4, 7), (4, 8), (2, 7), (2, 8)\}$$

Question 4:

State whether each of the following statements are true or false. If the statement is false, rewrite the given statement correctly.

- (i) If $P = \{m, n\}$ and $Q = \{n, m\}$, then $P \times Q = \{(m, n), (n, m)\}$
- (ii) If A and B are non-empty sets, then $(A \times B)$ is a non-empty set of ordered pairs such that $x \in A$ and $y \in B$.
- (iii) If $A = \{1, 2\}$, $B = \{3, 4\}$, then $A \times (B \cap \phi) = \phi$

Solution:

- (i) False, $P \times Q = \{(m, n), (m, m), (n, n), (n, m)\}$
- (ii) True
- (iii) True

Question 5:

If $A = \{-1, 1\}$, find $A \times A \times A$.

Solution:

It is known that for any non-empty set A, $A \times A \times A$ is defined as

$$A \times A \times A = \{(a, b, c) : a, b, c \in A\}$$

It is given that $A = \{-1, 1\}$

Therefore,

$$A \times A \times A = \{(-1, -1, -1), (-1, -1, 1), (-1, 1, -1), (-1, 1, 1), (1, -1, -1), (1, -1, 1), (1, 1, -1), (1, 1, 1)\}$$

Question 6:

If $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$. Find A and B.

Solution:

It is given that $A \times B = \{(a, x), (a, y), (b, x), (b, y)\}$

We know that the Cartesian product of two non-empty sets P and Q is defined as

$$P \times Q = \{(p, q) : p \in P, q \in Q\}$$

Therefore, A is the set of all first elements and B is the set of all second elements.

Thus, $A = \{a, b\}$ and $B = \{x, y\}$

Question 7:

Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$, $C = \{5, 6\}$ and $D = \{5, 6, 7, 8\}$. Verify that

- (i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.
- (ii) $A \times C$ is a subset of $B \times D$.

Solution:

- (i) To verify: $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have $B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\} = \phi$

$$\begin{aligned} LHS &= A \times (B \cap C) \\ &= A \times \phi \\ &= \phi \end{aligned}$$

Now,

$$\begin{aligned} A \times B &= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)\} \\ A \times C &= \{(1, 5), (1, 6), (2, 5), (2, 6)\} \end{aligned}$$

Therefore,

$$\begin{aligned} RHS &= (A \times B) \cap (A \times C) \\ &= \phi \end{aligned}$$

Therefore, L.H.S. = R.H.S

Hence, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

- (ii) To verify: $A \times C$ is a subset of $B \times D$

We have

$$A \times C = \{(1, 5), (1, 6), (2, 5), (2, 6)\}$$

$$B \times D = \left\{ (1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), \right. \\ \left. (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8) \right\}$$

We can observe that all the elements of set $A \times C$ are the elements of set $B \times D$.

Therefore, $A \times C$ is a subset of $B \times D$.

Question 8:

Let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write $A \times B$. How many subsets will $A \times B$ have? List them.

Solution:

$$A = \{1, 2\} \text{ and } B = \{3, 4\}$$

$$\text{Therefore, } A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

$$\text{Hence, } n(A \times B) = 4$$

We know that if C is a set with $n(C) = m$, then $n[P(C)] = 2^m$.

Therefore, the set $A \times B$ has $2^4 = 16$ subsets. These are

$$\left\{ \begin{aligned} &\phi, \{(1, 3)\}, \{(1, 4)\}, \{(2, 3)\}, \{(2, 4)\}, \\ &\{(1, 3), (1, 4)\}, \{(1, 3), (2, 3)\}, \{(1, 3), (2, 4)\}, \\ &\{(1, 4), (2, 3)\}, \{(1, 4), (2, 4)\}, \{(2, 3), (2, 4)\}, \\ &\{(1, 3), (1, 4), (2, 3)\}, \{(1, 3), (1, 4), (2, 4)\}, \\ &\{(1, 3), (2, 3), (2, 4)\}, \{(1, 4), (2, 3), (2, 4)\}, \{(1, 3), (1, 4), (2, 3), (2, 4)\} \end{aligned} \right\}$$

Question 9:

Let A and B be two sets such that $n(A) = 3$ and $n(B) = 2$. If $(x, 1), (y, 2), (z, 1)$ are in $A \times B$, find A and B , where x, y and z are distinct elements.

Solution:

It is given that $n(A) = 3$ and $n(B) = 2$; and $(x, 1), (y, 2), (z, 1)$ are in $A \times B$.

We know that

A = Set of first elements of the ordered pair elements of $A \times B$.

B = Set of second elements of the ordered pair elements of $A \times B$.

Therefore, x, y , and z are the elements of A ; 1 and 2 are the elements of B .

Since $n(A) = 3$ and $n(B) = 2$,

It is clear that $A = \{x, y, z\}$ and $B = \{1, 2\}$.

Question 10:

The Cartesian product $A \times A$ has 9 elements among which are found $(-1, 0)$ and $(0, 1)$. Find the set A and the remaining elements of $A \times A$.

Solution:

We know that if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

Therefore, $n(A \times A) = n(A) \times n(A)$

It is given that $n(A \times A) = 9$

$$n(A) \times n(A) = 9$$

Hence,

$$n(A) = 3$$

The ordered pairs $(-1, 0)$ and $(0, 1)$ are two of the nine elements of $A \times A$.

We know that $A \times A = \{(a, a) : a \in A\}$.

Therefore, $-1, 0$, and 1 are elements of A.

Since $n(A) = 3$, it is clear that $A = \{-1, 0, 1\}$.

The remaining elements of set $A \times A$ are $(-1, -1), (-1, 1), (0, -1), (0, 0), (1, -1), (1, 0)$, and $(1, 1)$

EXERCISE 2.2

Question 1:

Let $A = \{1, 2, 3, \dots, 14\}$. Define a relation R from A to A by $R = \{(x, y), 3x - y = 0; x, y \in A\}$. Write down its domain, co-domain and range.

Solution:

The relation R from A to A is given as $R = \{(x, y), 3x - y = 0; x, y \in A\}$.

Thus, $R = \{(x, y), 3x = y; x, y \in A\}$.

Therefore, $R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

Hence, Domain of $R = \{1, 2, 3, 4\}$

The whole set A is the co-domain of the relation R .

Therefore, Co-domain of $R = A = \{1, 2, 3, \dots, 14\}$

The range of R is the set of all second elements of the ordered pairs in the relation.

Therefore, Range of $R = \{3, 6, 9, 12\}$

Question 2:

Define a relation R on the set N of natural numbers by

$R = \{(x, y): y = x + 5, x \text{ is a natural number less than } 4; x, y \in N\}$.

Depict this relationship using roster form. Write down the domain and the range.

Solution:

$R = \{(x, y): y = x + 5, x \text{ is a natural number less than } 4; x, y \in N\}$

The natural numbers less than 4 are 1, 2, and 3.

Therefore, $R = \{(1, 6), (2, 7), (3, 8)\}$

The domain of R is the set of all first elements of the ordered pairs in the relation.

Hence, Domain of $R = \{1, 2, 3\}$.

The range of R is the set of all second elements of the ordered pairs in the relation.

Therefore, Range of $R = \{6, 7, 8\}$.

Question 3:

$A = \{1, 2, 3, 5\}$ and $B = \{4, 6, 9\}$. Define a relation R from A to B by

$R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$. Write R in roster form.

Solution:

$A = \{1, 2, 3, 5\}$, $B = \{4, 6, 9\}$ and $R = \{(x, y) : \text{the difference between } x \text{ and } y \text{ is odd}; x \in A, y \in B\}$

Therefore, $R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

Question 4:

The Fig2.7 shows a relationship between the sets P and Q . Write this relation

(i) in set-builder form (ii) in roster form.

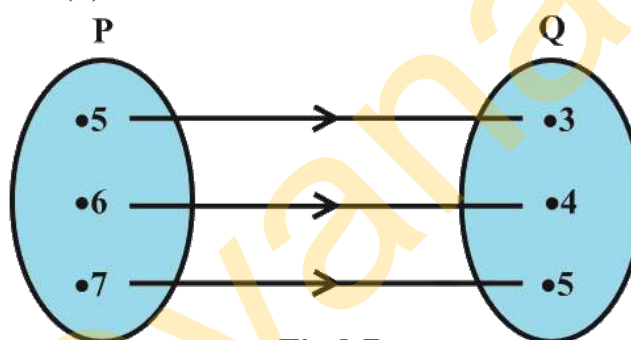


Fig 2.7

What is its domain and range?

Solution:

As per the Fig2.7, $P = \{5, 6, 7\}$ and $Q = \{3, 4, 5\}$

(i) $R = \{(x, y) : y = x - 2; x \in P\}$ or $R = \{(x, y) : y = x - 2, \text{ for } x = 5, 6, 7\}$

(ii) $R = \{(5, 3), (6, 4), (7, 5)\}$

Domain of $R = \{5, 6, 7\}$

Range of $R = \{3, 4, 5\}$

Question 5:

Let $A = \{1, 2, 3, 4, 6\}$. Let R be the relation on A defined by

$\{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$

- (i) Write R in roster form
- (ii) Find the domain of R
- (iii) Find the range of R.

Solution:

It is given that $A = \{1, 2, 3, 4, 6\}$ and $R = \{(a, b) : a, b \in A, b \text{ is exactly divisible by } a\}$

- (i) $R = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 6), (2, 2), (2, 4), (2, 6), (3, 3), (3, 6), (4, 4), (6, 6)\}$
- (ii) Domain of $R = \{1, 2, 3, 4, 6\}$
- (iii) Range of $R = \{1, 2, 3, 4, 6\}$

Question 6:

Determine the domain and range of the relation R defined by

$$R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}.$$

Solution:

It is given that $R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$

Therefore, $R = \{(0, 5), (1, 6), (2, 7), (3, 8), (4, 9), (5, 10)\}$

Hence,

$$\text{Domain of } R = \{0, 1, 2, 3, 4, 5\}$$

$$\text{Range of } R = \{5, 6, 7, 8, 9, 10\}$$

Question 7:

Write the relation $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$ in roster form.

Solution:

It is given that $R = \{(x, x^3) : x \text{ is a prime number less than } 10\}$

The prime numbers less than 10 are 2, 3, 5, and 7.

Therefore, $R = \{(2, 8), (3, 27), (5, 125), (7, 343)\}$

Question 8:

Let $A = \{x, y, z\}$ and $B = \{1, 2\}$. Find the number of relations from A to B.

Solution:

It is given that $A = \{x, y, z\}$ and $B = \{1, 2\}$.

Therefore, $A \times B = \{(x, 1), (x, 2), (y, 1), (y, 2), (z, 1), (z, 2)\}$

Since $n(A \times B) = 6$, the number of subsets of $(A \times B) = 2^6$.

Hence, the number of relations from A to B is 2^6 .

Question 9:

Let R be the relation on $\mathbf{Z}$ defined by $R = \{(a, b) : a, b \in \mathbf{Z}, a - b \text{ is an integer}\}$. Find the domain and range of R.

Solution:

It is given that $R = \{(a, b) : a, b \in \mathbf{Z}, a - b \text{ is an integer}\}$

It is known that the difference between any two integers is always an integer.

Therefore,

Domain of $R = \mathbf{Z}$

Range of $R = \mathbf{Z}$

EXERCISE 2.3

Question 1:

Which of the following relations are functions? Give reasons. If it is a function, determine its domain and range.

- (i) $\{(2,1), (5,1), (8,1), (11,1), (14,1), (17,1)\}$
- (ii) $\{(2,1), (4,2), (6,3), (8,4), (10,5), (12,6), (14,7)\}$
- (iii) $\{(1,3), (1,5), (2,5)\}$

Solution:

- (i) $\{(2,1), (5,1), (8,1), (11,1), (14,1), (17,1)\}$

Since 2, 5, 8, 11, 14, and 17 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = $\{2, 5, 8, 11, 14, 17\}$ and range = $\{1\}$

- (ii) $\{(2,1), (4,2), (6,3), (8,4), (10,5), (12,6), (14,7)\}$

Since 2, 4, 6, 8, 10, 12 and 14 are the elements of the domain of the given relation having their unique images, this relation is a function.

Here, domain = $\{2, 4, 6, 8, 10, 12, 14\}$ and range = $\{1, 2, 3, 4, 5, 6, 7\}$

- (iii) $\{(1,3), (1,5), (2,5)\}$

Since the same first element i.e., 1 corresponds to two different images i.e., 3 and 5, this relation is not a function.

Question 2:

Find the domain and range of the following real function:

- (i) $f(x) = -|x|$
- (ii) $f(x) = \sqrt{9-x^2}$

Solution:

- (i) $f(x) = -|x|, x \in \mathbf{R}$

We know that, $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$

Therefore, $f(x) = -|x| = \begin{cases} -x, & \text{if } x \geq 0 \\ x, & \text{if } x < 0 \end{cases}$

Since $f(x)$ is defined for $x \in \mathbf{R}$, the domain of $f = \mathbf{R}$

It can be observed that the range of $f(x) = -|x|$ is all real numbers except positive real numbers.

Therefore, the range of f is $f = (-\infty, 0]$

(ii) $f(x) = \sqrt{9 - x^2}$

Since $\sqrt{9 - x^2}$ is defined for all real numbers that are greater than or equal to -3 and less than or equal to 3 , the domain of $f(x)$ is $\{x : -3 \leq x \leq 3\}$ or $[-3, 3]$.

For any value of x such that $-3 \leq x \leq 3$, the value of $f(x)$ will lie between 0 and 3 .

Therefore, the range of $f(x)$ is $\{x : 0 \leq x \leq 3\}$ or $[0, 3]$

Question 3:

A function f is defined by $f(x) = 2x - 5$. Write down the values of

(i) $f(0)$ (ii) $f(7)$ (iii) $f(-3)$

Solution:

The given function is $f(x) = 2x - 5$.

Therefore,

(i) $f(0) = 2 \times 0 - 5 = 0 - 5 = -5$

(ii) $f(7) = 2 \times 7 - 5 = 14 - 5 = 9$

(iii) $f(-3) = 2 \times (-3) - 5 = -6 - 5 = -11$

Question 4:

The function ' t ' which maps temperature in degree Celsius into temperature in degree

Fahrenheit is defined by $t(C) = \frac{9C}{5} + 32$.

Find (i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) The value of C , when $t(C) = 212$.

Solution:

The given function is $t(C) = \frac{9C}{5} + 32$

Therefore,

$$(i) \quad t(0) = \frac{9 \times 0}{5} + 32 = 0 + 32 = 32$$

$$(ii) \quad t(28) = \frac{9 \times 28}{5} + 32 = \frac{252 + 160}{5} = \frac{412}{5} = 82.4$$

$$(iii) \quad t(-10) = \frac{9 \times (-10)}{5} + 32 = 9 \times (-2) + 32 = -18 + 32 = 14$$

(iv) It is given that

$$\begin{aligned} t(C) &= 212 \\ \Rightarrow \frac{9C}{5} + 32 &= 212 \\ \Rightarrow \frac{9C}{5} &= 212 - 32 \\ \Rightarrow C &= \frac{180 \times 5}{9} = 100 \end{aligned}$$

Thus, the value of 't', when $t(C) = 212$ is 100.

Question 5:

Find the range of each of the following functions.

$$(i) \quad f(x) = 2 - 3x, x \in \mathbf{R}, x > 0.$$

$$(ii) \quad f(x) = x^2 + 2, x \text{ is a real number.}$$

$$(iii) \quad f(x) = x, x \text{ is a real number.}$$

Solution:

$$(i) \quad f(x) = 2 - 3x, x \in \mathbf{R}, x > 0.$$

The values of $f(x)$ for various values of real numbers $x > 0$ can be written in the tabular form as

x	0.01	0.1	0.9	1	2	2.5	4	5	...
$f(x)$	1.97	1.7	-0.7	-1	-4	-5.5	-10	-13	...

Thus, it can be clearly observed that the range of f is the set of all real numbers less than 2. i.e., range of $f = (-\infty, 2)$

Alternative Method:

Let $x > 0$

$$\Rightarrow 3x > 0$$

$$\Rightarrow 2 - 3x < 2$$

$$\Rightarrow f(x) < 2$$

Therefore, Range of $f = (-\infty, 2)$

(ii) $f(x) = x^2 + 2, x$ is a real number.

The values of $f(x)$ for various values of real numbers x can be written in the tabular form as

x	0	± 0.3	± 0.8	± 1	± 2	± 3	...
$f(x)$	2	2.09	2.64	3	6	11	...

Thus, it can be clearly observed that the range of f is the set of all real numbers greater than 2. i.e., range of $f = [2, \infty)$

Alternative Method:

Let x be any real number i.e., $x^2 \geq 0$.

Accordingly,

$$x^2 \geq 0$$

$$\Rightarrow x^2 + 2 \geq 0 + 2$$

$$\Rightarrow x^2 + 2 \geq 2$$

$$\Rightarrow f(x) \geq 2$$

Therefore, Range of $f = [2, \infty)$

(iii) $f(x) = x, x$ is a real number

It is clear that, the range of f is the set of all real numbers.

Therefore, Range of $f = \mathbf{R}$.

MISCELLANEOUS EXERCISE

Question 1:

The relation f is defined by $f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$

The relation g is defined by $g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$

Show that f is a function and g is not a function.

Solution:

The relation f is defined as $f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$

It can be observed that for

$$0 \leq x < 3, f(x) = x^2 \text{ and}$$

$$3 < x \leq 10, f(x) = 3x$$

Also, at $x = 3$

$$\begin{aligned} f(x) &= 3^2 \\ &= 9 \end{aligned}$$

and

$$\begin{aligned} f(x) &= 3 \times 3 \\ &= 9 \end{aligned}$$

i.e., at $x = 3, f(x) = 9$

Therefore, for $0 \leq x \leq 10$, the images of $f(x)$ are unique.
Thus, the given relation is a function.

The relation g is defined as $g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$

It can be observed that for

$$0 \leq x \leq 2, g(x) = x^2 \text{ and}$$

$$2 \leq x \leq 10, g(x) = 3x$$

Also, at $x = 2$

$$\begin{aligned}g(x) &= 2^2 \\ &= 4\end{aligned}$$

and

$$\begin{aligned}g(x) &= 3 \times 2 \\ &= 6\end{aligned}$$

Hence, element 2 of the domain of the relation g corresponds to two different images i.e., 4 and 6.

Hence, this relation is not a function.

Thus, f is a function and g is not a function.

Question 2:

If $f(x) = x^2$, find $\frac{f(1.1) - f(1)}{(1.1 - 1)}$.

Solution:

It is given that $f(x) = x^2$

Therefore,

$$\begin{aligned}\frac{f(1.1) - f(1)}{(1.1 - 1)} &= \frac{(1.1)^2 - (1)^2}{(1.1 - 1)} \\ &= \frac{1.21 - 1}{0.1} \\ &= \frac{0.21}{0.1} \\ &= 2.1\end{aligned}$$

Question 3:

Find the domain of the function $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Solution:

The given function is $f(x) = \frac{x^2 + 2x + 1}{x^2 - 8x + 12}$.

$$\begin{aligned}f(x) &= \frac{x^2 + 2x + 1}{x^2 - 8x + 12} \\ &= \frac{x^2 + 2x + 1}{(x - 6)(x - 2)}\end{aligned}$$

It can be seen that, the function f is defined for all real numbers except at $x = 6$ and $x = 2$.

Hence, the domain of the function f is set of real numbers except 6 and 2. i.e., $\mathbf{R} - \{2, 6\}$.

Question 4:

Find the domain and the range of the real function f defined by $f(x) = \sqrt{(x-1)}$.

Solution:

The given real function is $f(x) = \sqrt{(x-1)}$.

It can be seen that $\sqrt{(x-1)}$ is defined for $x \geq 1$.

Therefore, the domain of f is the set of all real numbers greater than or equal to 1 i.e., the domain of $f = [1, \infty)$.

As

$$\Rightarrow x \geq 1$$

$$\Rightarrow (x-1) \geq 0$$

$$\Rightarrow \sqrt{(x-1)} \geq 0$$

Therefore, the range of f is the set of all real numbers greater than or equal to 0 i.e., the range of $f = [0, \infty)$.

Question 5:

Find the domain and the range of the real function f defined by $f(x) = |x-1|$.

Solution:

The given real function is $f(x) = |x-1|$

It is clear that $|x-1|$ is defined for all real numbers.

Therefore, Domain of $f = \mathbf{R}$

Also, for $x \in \mathbf{R}$, $|x-1|$ assumes all real numbers.

Hence, the range of f is the set of all non-negative real numbers.

Question 6:

Let $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$ be a function from $\mathbf{R}$ into $\mathbf{R}$. Determine the range of f .

Solution:

It is given that $f = \left\{ \left(x, \frac{x^2}{1+x^2} \right) : x \in \mathbf{R} \right\}$

Therefore, $\left\{ (0,0), \left(\pm 0.5, \frac{1}{5} \right), \left(\pm 1, \frac{1}{2} \right), \left(\pm 1.5, \frac{9}{13} \right), \left(\pm 2, \frac{4}{5} \right), \left(\pm 3, \frac{9}{10} \right), \left(\pm 4, \frac{16}{17} \right), \dots \right\}$

The range of f is the set of all second elements. It can be observed that all these elements are greater than or equal to 0 but less than 1 (denominator is greater than numerator).

Thus, range of $f = [0, 1)$

Question 7:

Let $f, g : \mathbf{R} \rightarrow \mathbf{R}$ be defined, respectively by $f(x) = x+1$, $g(x) = 2x-3$. Find $f+g$, $f-g$ and $\frac{f}{g}$.

Solution:

Since $f, g : \mathbf{R} \rightarrow \mathbf{R}$ is defined as $f(x) = x+1$, $g(x) = 2x-3$
Hence,

$$\begin{aligned} (f+g)(x) &= f(x) + g(x) \\ &= (x+1) + (2x-3) \\ &= 3x-2 \end{aligned}$$

Therefore, $(f+g)(x) = 3x-2$

Now,

$$\begin{aligned} (f-g)(x) &= f(x) - g(x) \\ &= (x+1) - (2x-3) \\ &= x+1-2x+3 \\ &= -x+4 \end{aligned}$$

Therefore, $(f - g)(x) = -x + 4$

Now,

$$\begin{aligned}\left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)}, g(x) \neq 0, x \in R \\ &= \frac{x+1}{2x-3}, 2x-3, x \neq \frac{3}{2}\end{aligned}$$

Therefore, $\left(\frac{f}{g}\right)(x) = \frac{x+1}{2x-3}, x \neq \frac{3}{2}$

Question 8:

Let $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ be a function from $\mathbf{Z}$ to $\mathbf{Z}$ defined by $f(x) = ax + b$, for some integers a, b . Determine a, b .

Solution:

It is given that $f = \{(1,1), (2,3), (0,-1), (-1,-3)\}$ and $f(x) = ax + b$

$$(1, 1) \in f \Rightarrow f(1) = 1 \Rightarrow a \times 1 + b = 1$$

$$\Rightarrow a + b = 1$$

$$(0, -1) \in f \Rightarrow f(0) = -1 \Rightarrow a \times 0 + b = -1$$

$$\Rightarrow b = -1$$

On substituting $b = -1$ in $a + b = 1$, we obtain

$$a + (-1) = 1$$

$$a = 1 + 1$$

$$= 2$$

Thus, the respective values of a and b are 2 and -1 .

Question 9:

Let R be a relation from $\mathbf{N}$ to $\mathbf{N}$ defined by $R = \{(a, b) : a, b \in \mathbf{N} \text{ and } a = b^2\}$. Are the following true?

(i) $(a, a) \in R$, for all $a \in \mathbf{N}$

(ii) $(a, b) \in R$, implies $(b, a) \in R$

(iii) $(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$

Justify your answer in each case.

Solution:

It is given that $R = \{(a, b) : a, b \in \mathbb{N} \text{ and } a = b^2\}$

- (i) It can be seen that $2 \in \mathbb{N}$; however, $2 \neq 2^2 = 4$

Therefore, the statement “ $(a, a) \in R$, for all $a \in \mathbb{N}$ ” is not true.

- (ii) It can be seen that $(9, 3) \in R$ because $9, 3 \in \mathbb{N}$ and $9 = 3^2$.

Now, $3 \neq 9^2 = 81$; therefore, $(3, 9) \notin R$

Therefore, the statement “ $(a, b) \in R$, implies $(b, a) \in R$ ” is not true.

- (iii) It can be seen that $(9, 3) \in R, (16, 4) \in R$ because $9, 3, 16, 4 \in \mathbb{N}$ and $9 = 3^2, 16 = 4^2$

Now, $9 \neq 4^2 = 16$; therefore, $(9, 4) \notin R$.

Therefore, the statement “ $(a, b) \in R, (b, c) \in R$ implies $(a, c) \in R$ ” is not true.

Question 10:

Let $A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$. Are the following are true?

- (i) f is a relation from A to B (ii) f is a function from A to B.

Justify your answer in each case.

Solution:

It is given that $A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$ and $f = \{(1, 5), (2, 9), (3, 1), (4, 5), (2, 11)\}$.
Therefore,

$$A \times B = \left\{ \begin{array}{l} (1, 1), (1, 5), (1, 9), (1, 11), (1, 15), (1, 16), (2, 1), (2, 5), (2, 9), \\ (2, 11), (2, 15), (2, 16), (3, 1), (3, 5), (3, 9), (3, 11), (3, 15), \\ (3, 16), (4, 1), (4, 5), (4, 9), (4, 11), (4, 15), (4, 16) \end{array} \right\}$$

- (i) A relation from a non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$.

It is observed that f is a subset of $A \times B$.

Thus, f is a relation from A to B.

- (ii) Since the same first element i.e., 2 corresponds to two different images i.e., 9 and 11.

Thus, f is not a function from A to B.

Question 11:

Let f be the subset of $\mathbf{Z} \times \mathbf{Z}$ defined by $f = \{(ab, a+b) : a, b \in \mathbf{Z}\}$. Is f a function from $\mathbf{Z}$ to $\mathbf{Z}$? Justify your answer.

Solution:

The relation f is defined as $f = \{(ab, a+b) : a, b \in \mathbf{Z}\}$.

We know that a relation f from a set A to a set B is said to be a function if every element of set A has unique images in set B.

Since, $2, 6, -2, -6 \in \mathbf{Z}, (2 \times 6, 2+6), (-2 \times -6, -2+(-6)) \in f$
i.e., $(12, 8), (12, -8) \in f$

It can be seen that the same first element i.e., 12 corresponds to two different images i.e., 8 and -8.

Thus, relation f is not a function.

Question 12:

Let $A = \{9, 10, 11, 12, 13\}$ and let $f : A \rightarrow \mathbf{N}$ be defined by $f(n) =$ the highest prime factor of n . Find the range of f .

Solution:

It is given that $A = \{9, 10, 11, 12, 13\}$ and $f : A \rightarrow \mathbf{N}$ is defined by

$f(n) =$ the highest prime factor of n

Hence,

Prime factor of 9 = 3

Prime factor of 10 = 2, 5

Prime factor of 11 = 11

Prime factor of 12 = 2, 3

Prime factor of 13 = 13

Therefore,

$f(9) =$ the highest prime factor of 9 = 3

$f(10) =$ the highest prime factor of 10 = 5

$f(11) =$ the highest prime factor of 11 = 11

$f(12) =$ the highest prime factor of 12 = 3

$f(13)$ = the highest prime factor of $13 = 13$

The range of f is the set of all $f(n)$, where $n \in A$.

Therefore, Range of $f = \{3, 5, 11, 13\}$

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